3.3 / 75

How does differentiation apply to piecewise functions ?

$f(x)=\begin{cases} x^2+x+1, & x\le 1 \\ 3x, & x>1 \end{cases}$. Show f is continuous at $x = 1$.

This is stuff we've done before. We use the three-pronged definition of continuity:

1)
$$\lim_{x\to 1^+} f(x)=\lim_{x\to 1^+} 3x=3$$
$$\lim_{x\to 1^-} f(x)=\lim_{x\to 1^-} x^2+x+1=3$$
$$\therefore \lim_{x\to 1} f(x)=3$$

2) $f(1)$ is defined and = 3

3) $\lim_{x\to 1} f(x)=f(1)=3$

And so the function is continuous.

Now … is the function ***differentiable*** at $x = 1$? We **could** use the limit definition of derivative from both sides to see ….. but let's think about how we would apply the rules …. A derivative rule (like the power rule; or the rule that the derivative of *sin x* is *cos x*) applies everywhere that the derivative (a two sided limit) exists. How does the two sided limit exist? It must have "room" on both sides for both one-sided limits.

Therefore – we can use the derivative "rule" on the entire domain of that "piece" – excluding the endpoints of a closed interval (no other side …..).

We have (initially) the following then:

$f'(x)=\begin{cases} 2x+1, & x<1 \\ 3, & x>1 \end{cases}$. You see that we eliminated the endpoint ($x = 1$) from the domain.

Now – what happens **AT** $x = 1$? Well … the derivative would be a two sided limit, so **we can check the derivative rule from each side.** If they agree, then the two-sided limit would exist, AND the derivative would exist.

So $\lim_{x\to 1^-} f'(x)=2(1)+1=3$ for $x < 1$. $\lim_{x\to 1^+} f'(x)=3$ for $x > 1$. Since those 'one sided' derivatives are **equal**, then the two sided limit exists, and we conclude that:

$f(x)$ is differentiable at $x = 1$, and $f'(1)=3$.

What does this mean graphically (or pictorially)? At the place where the two pieces are "glued" together ($x = 1$), the function is continuous (the pieces meet), **and** the slopes match (the function is differentiable), and so no one will be able to tell 'where the seam is'…..

3.4 / 19

To find $\frac{d^2y}{dx^2}$ means to find the second derivative (alternative Leibniz notation ….). To do this, we take the derivative of the first derivative. So ….. (and note we need to use the product rule ….)

$$\frac{d}{dx}(x\cos x) = (1)(\cos x) + (x)(-\sin x) = \cos x - x\sin x$$
$$\frac{d^2}{dx^2}(x\cos x) = \frac{d}{dx}\left(\frac{d}{dx}(x\cos x)\right) = \frac{d}{dx}(\cos x - x\sin x) =$$
$$(-\sin x) - \left[(1)(\sin x) + (x)(\cos x)\right] = -2\sin x - x\cos x$$

You should be able to figure out where I used the product rule (I used it twice!). If in doubt, ask ☺

Hope this helps!

3.5 / 7

Use the **chain rule** to find the derivative of $y = (x^3 + 2x)^{37}$. If I were to evaluate this function at $x = 1$, say, I would first evaluate $(x^3 + 2x)$ at $x = 1$, and then I would raise *that* number to the 37th power. That tells us that $(x^3 + 2x)$ is the **inside** function, and x^{37} is the outside function.

So .. how do I use the chain rule? First I find the derivative of x^{37} which equals $37x^{36}$ and then "evaluate that" (or compose that with ….) $(x^3 + 2x)$.

But before I'm done, I have to multiply that by the derivative of the inside function. The derivative of $(x^3 + 2x)$ is $(3x^2 + 2)$. So … my final answer is: $37(x^3 + 2x)^{36} \bullet (3x^2 + 2)$.

If you don't see the considerable value of this rule (opposite for example multiplying out the original function and then using the power rule), then perhaps you should multiply out ……

Make sure you understand each piece ☺